Dec 2012

This question paper contains 2 printed pages. Your Roll No. ....

Sl. No. Of Ques. Paper	:	8420C
Unique Paper Code	:	222501
Name of the Paper	:	PHHT- 515 : Mathematical Physics - V
Name of the Course	:	B.Sc. (Hons) Physics Part III
Semester	:	V
Duration	:	3 hours
Maximum Marks	:	75

Do five questions in all. All questions carry equal marks. Question No. 1 is compulsory. Do two questions from each Section.

Q1. Do any *five* : (15)

a) Fourier Transform of a Gaussian function is a Gaussian function.

b) Find L(F(t)) where

$F(t) = \cos\left(t - \frac{2\pi}{3}\right) u\left(t - \frac{2\pi}{3}\right)$ and

$u\left(t - \frac{2\pi}{3}\right)$ is the unit step function.

c) Find $L(t^n)$ for n as a positive integer.

d) Show that $L(\delta(t)) = 1$ where $\delta(t)$ is the Dirac Delta function.

e) Show that div (curl F) = 0 using tensors.

f) Show that velocity and acceleration are contravariant vectors.

g) Show that $\vec{A} \times \vec{B}$ transforms like tensor of rank one.

Section A

Q2. (10+5)

a) Find the Fourier sine transform of

$f(t) = e^{-pt}$ p > 0 and

Evaluate the integral

$$\int_0^\infty \frac{\omega \sin \omega t}{\omega^{2+}p^2} d\omega$$

b) Show that $\int_{-\infty}^{\infty} f(x)\delta'(x)dx = -f'(0)$.

Q3. (5+5+5)

a) Prove that

$$L\left(\frac{1}{t} f(t)\right) = \int_s^\infty f(s)ds$$

b) Using convolution theorem for Laplace transforms, find

$$L^{-1}\left(\frac{1}{s\,(s^2 + a^2)}\right)$$

c) Show that the derivative of unit step function is Dirac Delta Function.

Q4. (12+3)

a) Solve the given coupled differential equations using Laplace Transforms

$$\frac{dx}{dt} = 2x - 3y$$
$$\frac{dy}{dt} = y - 2x$$

subject to initial conditions $x(0) = 8,\ y(0) = 3.$

b) Find out

$$L\left(\int_0^t \frac{\sin u}{u}\, du\right)$$

Section B

Q5. (5+10)

a) Given vector

$$\vec{U} = (x, \quad x + y, \quad x + y + z)$$

Find the second order anti-symmetric tensor associated with it.

b) Show that

$$\epsilon_{ijk}\,\epsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

Hence prove

$$\epsilon_{ijk}\,\epsilon_{ijk} = 6$$

Q6. (10+5)

a) Define the Pure Strain Tensor e_{ij} . Establish that it is a symmetric tensor of order 2. Also give the physical significance of its components e_{11} and e_{12} .

b) Define Quotient Law.
Let $A(i, j, k)$ be a set of N^3 functions whose inner product with an arbitrary tensor B^{jk} yields a tensor C^i . What can you conclude about $A(i, j, k)$?

Q7. (12+3)

a) The length ds of a line element in a 2-dimensional suface $\theta, \emptyset$ is given by
$$ds^2 = R^2\, d\theta^2 + R^2 \sin^2\theta\, d\emptyset^2 \qquad \text{with } R = \text{constant.}$$
Find all the components of the metric tensor $g_{\mu\nu}$ and the Christoffel symbols of fi kind for this surface.

b) Show that $A^\mu B_\mu$ is invariant.

This question paper contains 4 printed pages]

24 NOV 2014

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 925

Unique Paper Code : 222501 E

Name of the Paper : **Mathematical Physics—V(PHHT-515)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do *Five* questions in all.

Question No. **1** is compulsory.

Do **2** questions from each Section.

1. Answer any *five* questions :

(*a*) Find the Fourier transform of the function :

$$f(x) = \frac{1}{\varepsilon} \text{ for } |x| \leq \varepsilon$$
$$f(x) = 0 \text{ for } |x| > \varepsilon$$

(*b*) If $F(\alpha)$ is the Fourier transform of $f(x)$, show that the Fourier transform of $f(x-a)$ is $e^{i\alpha a} F(\alpha)$.

P.T.O.

(*c*) Using Convolution theorem, evaluate :

$$L^{-1}\left\{\frac{1}{s^2(s-1)}\right\}$$

(*d*) Prove that :

$$L\{t\ f\ (t)\} = -\frac{dF(s)}{ds},$$

where

F(*s*) is the Laplace transform of (*t*).

(*e*) Show that the Dirac delta function :

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikx}\, dk.$$

(*f*) Given a vector = $7i + 3j + 4k$, find the components of the associated second rank anti-symmetric tensor.

(*g*) Prove that the contraction of the outer product of the tensors A^{μ} and B_{ν} is invariant. 3×5=15

Section A

2. (*a*) State and prove the Fourier Integral Theorem.

(*b*) Find $L\{t^2 \cos at\}$. 10,5

(a) Using Laplace transform, solve the following differential equation :

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5\sin t$$

where,

$$y(0) = y'(0) = 0$$

(b) Find the Fourier transform of :

$$f(x) = 1 - x^2 \text{ for } |x| < 1$$

$$f(x) = 0 \text{ for } |x| > 1$$

9,6

(a) Show that :

$$\int_0^\infty \cos x^2 \, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$$

using Laplace transform.

(b) Given :

$$f(x) = 1 \text{ for } |x| < a$$

and $f(x) = 0$ for $|x| > a$

evaluate the following integral using Parseval's identity :

8,7

$$\int_0^\infty \frac{\sin^2 ka}{k^2} \, dk.$$

P.T.O.

Section B

5. (*a*) Prove that the Moment of Inertia is a second order symmetric tensor and express it in the matrix form.

(*b*) Given that :

$$ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$$

determine the metric tensor g_{pq} and express it as a matrix. 10,5

6. (*a*) Prove the following identity using tensors :

$$\nabla (A \cdot B) = (B \cdot \nabla)A + (A \cdot \nabla)B + B\times(\nabla \times A) + A\times(\nabla \times B)$$

(*b*) Show that the Kronecker delta is an isotropic second order symmetric tensor. 8,7

7. (*a*) Show that the covariant derivative of a covariant vector is a covariant tensor of rank two.

(*b*) Define alternating tensor and show that :

(*i*) It is an anti-symmetric tensor of order three

(*ii*) It is an isotropic tensor. 10,5

This question paper contains 2 printed pages.

Your Roll No. ....

F

Sl. No. of Ques. Paper	: 5767
Unique Paper Code	: 222501
Name of Paper	: Mathematical Physics – V (PHHT-515)
Name of Course	: B.Sc. (Hons.) Physics
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do five questions in all. Question No. 1 is compulsory.
Do two questions from each Section.

1. Do any ***five*** questions :

(a) If F(k) is the Fourier transform of f(x), find the Fourier transform of f(x) sin ax.

(b) Find the Fourier transform of the Dirac-Delta function $\delta(x-a)$, where a is positive real constant.

(c) If $L\{F(t)\} = f(s)$, find $L\{F(a\,t)\}$.

(d) Find the Laplace transform of unit step function $U(t-a)$, where a is positive real constant.

(e) Prove that $x\delta(x) = 0$.

(f) Show that $\delta_{ik} = a_{ij}\, a_{kj}$, where the symbols have their usual meaning.

(g) Show that every second order tensor can be expressed as the sum of a symmetric and skew-symmetric tensor.

(h) If $A_{ij}B_i$ is a vector, where B_i is any arbitrary vector, then prove that A_{ij} is a tensor of order two. (3 × 5 = 15)

Section A

2. (a) Find Fourier transform of the function:

$$f(x) = \begin{cases} 1 & ; |x| < a \\ 0 & ; |x| > a \end{cases}$$

and hence evaluate the integrals

$$\int_0^\infty \frac{\sin ax \cos \omega x}{x}\, dx \quad \text{and} \quad \int_0^\infty \frac{\sin x}{x}\, dx$$ (8)

Turn over

(b) Show that Fourier transform of

$$f(x) = e^{-x^2/2} \text{ is given by } g(p) = e^{-p^2/2}. \quad (7)$$

3. (a) Show that $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\} = \ln\left(\frac{s+b}{s+a}\right)$. (5)

(b) Using Laplace transform, solve the differential equation:

$$\frac{d^2Y}{dt^2} + 8\frac{dY}{dt} + 25\,Y = 150 \quad (10)$$

Given $Y(0) = \dot{Y}(0) = 0$

4. (a) Using Convolution theorem for Laplace transforms, find

$$L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\} \quad (8)$$

(b) Find $f(x)$ if its Fourier cosine transform is $\frac{1}{1+k^2}$. (7)

Section B

5. (a) Prove that

(i) $\delta_{ik} \in_{ikm} = 0$

(ii) $\in_{iks} \in_{mks} = 2\,\delta_{im}$ (3, 5)

(b) If T_{ij} is a skew-symmetric tensor of rank two, then prove that:

$$(\delta_{ij}\delta_{lk} + \delta_{il}\delta_{jk})T_{ik} = 0 \quad (3)$$

(c) Show that the gradient of a vector is a tensor of rank 2. (4)

6. (a) What do you understand by an isotropic tensor? Show that the only isotropic tensor of order 3 is the scalar multiple of alternating tensor $\in_{ikm}$. (1, 7)

(b) Using tensors, prove the following identities:

(i) $\vec{\nabla} \bullet (\vec{A} \times \vec{B}) = (\vec{\nabla} \times \vec{A}) \bullet \vec{B} - (\vec{\nabla} \times \vec{B}) \bullet \vec{A}$

(ii) $\vec{\nabla} \times (\vec{A} \times \vec{B}) = \vec{A}(\vec{\nabla} \bullet \vec{B}) - \vec{B}(\vec{\nabla} \bullet \vec{A}) + (\vec{B} \bullet \vec{\nabla})\vec{A} - (\vec{A} \bullet \vec{\nabla})\vec{B}$

(3, 4)

7. (a) If $ds^2 = 5(dx^1)^2 + 4(dx^2)^2 - 6\,dx^1 dx^2$, find the matrices

(i) (g_{ij}),

(ii) (g^{ij}), and

(iii) the product of (g_{ij}) and (g^{ij}). (2, 4, 2)

(b) Prove that:

(i) $[p\ m, q] + [q\ m, p] = \frac{\partial}{\partial x^q}(g_{pq})$,

(ii) $[p\ q, r] = g_{rs}\left\{ {s \atop p\ q} \right\}$.

s question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

No. of Question Paper : **852**

Unique Paper Code : **222501** **G**

Name of the Paper : **Mathematical Physics—V (PHHT-515)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do *five* questions in all.

Question No. **1** is compulsory.

Do **2** questions from each Section.

1. Do any *five* questions :

(*a*) If $F(k)$ is the Fourier transform of $f(x)$, find the Fourier transform of $f(ax)$.

(*b*) Find the Fourier transform of $e^{-|t|}$.

(*c*) If

$$L\{F(t)\} = f(s),$$

find $L\left\{F\left(\frac{t}{a}\right)\right\}$.

(*d*) Find the Laplace transform of the Dirac delta function $\delta(t-a)$, where a is positive real constant.

P.T.O.

(*e*) Show that :

$$\delta'(x) = -\frac{\delta(x)}{x}.$$

(*f*) Show that gradient of a scalar function is a tensor of order one.

(*g*) Find the second order anti-symmetric tensor associated with the vector :

$$2\hat{i} - 3\hat{j} + \hat{k}\cdot$$

(*h*) Prove that product of tensors of rank one is a tensor of rank two. 5×3=15

Section A

2. (*a*) Find Fourier sine transform of e^{-x} and hence prove that : 4,4

$$\int_0^\infty \frac{t \sin tx}{1+x^2}\, dt = \frac{\pi}{2} e^{-x}.$$

(*b*) State and prove Convolution theorem for Fourier transforms. 2,5

3. (*a*) Find : 5

$$L^{-1}\left\{\frac{1}{s^2(s^2+a^2)}\right\}.$$

(*b*) Using Laplace transforms, solve the following coupled differential equations :

$$\frac{dX}{dt} + Y = 0, \qquad \frac{dY}{dt} - X = 0$$

under the condition $X(0) = 1$, $Y(0) = 0$. 10

(*a*) For the function :

$$G(t) = \begin{cases} e^{-xt}\phi(t) & ; t < 0 \\ 0 & ; t > 0 \end{cases}$$

Find the relation between Fourier transform of $G(t)$ and Laplace transform of $\phi(t)$. 6

(*b*) If $F(t)$ is a periodic function of period T, find its Laplace transform. 5

(*c*) Prove that :

$$\delta(ax) = \frac{\delta(x)}{|a|},$$

where $a > 0$. 4

Section B

(*a*) Derive an expression for the Moment of Inertia tensor. Prove that it is a symmetric tensor and it transforms like a second order tensor. 5,2,3

(*b*) Show that : 5

$$\epsilon_{iks}\,\epsilon_{mps} = \delta_{im}\,\delta_{kp} - \delta_{ip}\,\delta_{km}.$$

(*a*) Define Kronecker-Delta function. Show that it is : 1,2,2

(*i*) an isotropic tensor

(*ii*) a symmetric tensor of order 2.

P.T.O.

(*b*) Using tensors, prove the following identities : 5,5

(*i*) $\vec{\nabla} \times \left(\phi \vec{A}\right) = \phi\left(\vec{\nabla} \times \vec{A}\right) + \left(\vec{\nabla}\phi\right) \times \vec{A}$

(*ii*) $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{A}\right) = \vec{\nabla}\left(\vec{\nabla} . \vec{A}\right) - \nabla^2 \vec{A}$.

7. (*a*) If

$$ds^2 = 3(dx^1)^2 + 5(dx^2)^2 - 4dx^1dx^2,$$

find the matrices :

(*i*) (g_{ij}),

(*ii*) (g^{ij}), and

(*iii*) the product of (g_{ij}) and (g^{ij}). 2,4,2

(*b*) Prove that : 7

$$\left\{ \begin{matrix} p \\ p\ q \end{matrix} \right\} = \frac{\partial}{\partial x^q} \ln \sqrt{g}.$$

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5782 F Your Roll No....................

Unique Paper Code : 222401

Name of the Paper : Mathematical Physics – IV (PHHT - 411)

Name of the Course : B. Sc. (Hons.) Physics

Semester : IV

0 9 MAY 2016

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all taking at least **one** question from each section.
3. **All** questions carry equal marks.

Section A

1. (a) Prove that the set Q_{-1} of all rational numbers other than –1 with the binary operation * defined by

$$a * b = a + b + ab$$

form a group.

OR

Determine whether or not W is a subspace of $\mathbf{R}^3$ where W consists of all vectors (a, b, c) in $\mathbf{R}^3$ such that

$$b = a^2.$$ (7)

(b) Find a basis and the dimension of solution space W of the following homogeneous system:

$$x + 2y + 2z - s + 3t = 0$$
$$x + 2y + 3z + s + t = 0$$
$$3x + 6y + 8z + s + 5t = 0$$ (8)

P.T.O.

2. (a) Linear Transformation T on $\mathbf{R}^3$ of all ordered triples of real numbers is defined by

$$T\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix},\quad T\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}3\\1\\2\end{bmatrix}\quad \text{and} \quad T\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$$

Compute $T\begin{bmatrix}4\\0\\5\end{bmatrix}$ (5)

(b) Linear transformation T on $\mathbf{R}^3$ is defined as

$$T\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2x-3y+4z\\5x-2y+2x\\4x+7y\end{bmatrix}.$$

Find the matrix representation of T relative to

(i) the standard basis $\left\{e_1=\begin{bmatrix}1\\0\\0\end{bmatrix}, e_2=\begin{bmatrix}0\\1\\0\end{bmatrix}, e_3=\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$.

(ii) the basis $\left\{\alpha_1=\begin{bmatrix}1\\1\\1\end{bmatrix}, \alpha_2=\begin{bmatrix}1\\1\\0\end{bmatrix}, \alpha_3=\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$. (10)

Section B

3. (a) Verify Cayley – Hamilton theorem for the matrix

$$A=\begin{bmatrix}3 & -4\\1 & 5\end{bmatrix}.$$

Hence find A^{-1}. (5)

(b) If A and B are Hermitian matrices, show that

AB + BA is Hermitian and AB − BA is skew-Hermitian. (5)

(c) If H is a Hermitian matrix and I is Identity matrix, show that

$$(I - iH)(I + iH)^{-1}$$

is a Unitary matrix.

$$(here\ ,\ i = \sqrt{-1})$$ (5)

4. (a) Find the eigenvalues and eigenvectors of the matrix :

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

Can A be diagonalized ? If yes, find a diagonalizing matrix P and verify that P diagonalizes A. (10)

(b) Determine 2^A if

$$A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$$ (5)

. (a) Solve the coupled differential equations

$$\ddot{y} = -5y + 2z$$

$$\ddot{z} = 2y - 2z$$

where, $y(0) = 1,\ z(0) = 2,\ \dot{y}(0) = 2,\ \dot{z}(0) = 1.$ (10)

(b) Write the symmetric coefficient matrix of the following quadratic form:

$$-3x_1^2 - x_2^2 - 5x_3^2 + 6x_1x_2 + 4x_1x_3$$ (5)

Section C

Write down the two dimensional wave equation in plane polar coordinates and solve it for a circular membrane of radius a specifying the relevant boundary and initial conditions. (15)

P.T.O.

7. Solve one – dimensional heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad 0 \le x \le L;\ t > 0$$

under the boundary conditions: $u(0, t) = 0$ and $u(L, t) = 0$

and initial condition: $u(x, 0) = \frac{h}{L} x$. (15)

(here, c and h are constants)

8. Write down three dimensional Laplace's equation in cylindrical (ρ, ϕ, z) as well as in spherical (r, θ, ϕ) coordinates. Obtain the solution of Laplace's equation in cylindrical coordinates which are independent of z. (15)